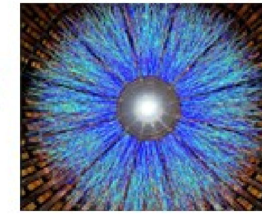
BERKELEY LAB
School of Collective Dynamics in High Energy Collisions
June 7 - 11, 2010



Nonequilibrium field theory and its applications

Christoph Herold

HGS-HIRe for FAIR
Helmholtz Graduate School for Hadron and Ion Research

GOETHE UNIVERSITÄT
FRANKFURT AM MAIN

Outline

- Motivation
- The closed time path formalism
- Green's functions
- Applications

Motivation

- Imaginary-time formalism:
  - t traded in for T
  - Static, equilibrium properties
- Study of dynamical systems:
  - Evolution of early universe
  - Chiral symmetry breaking

Closed time path formalism

$$\rho(t) = \sum_n p_n |\psi_n(t)> <\psi_n(t)|$$

Quantum Liouville equation

$$i \frac{\partial \rho(t)}{\partial t} = [H, \rho(t)]$$

General solution: $\rho(t) = U(t,0)\,\rho(0)\,U(0,t)$

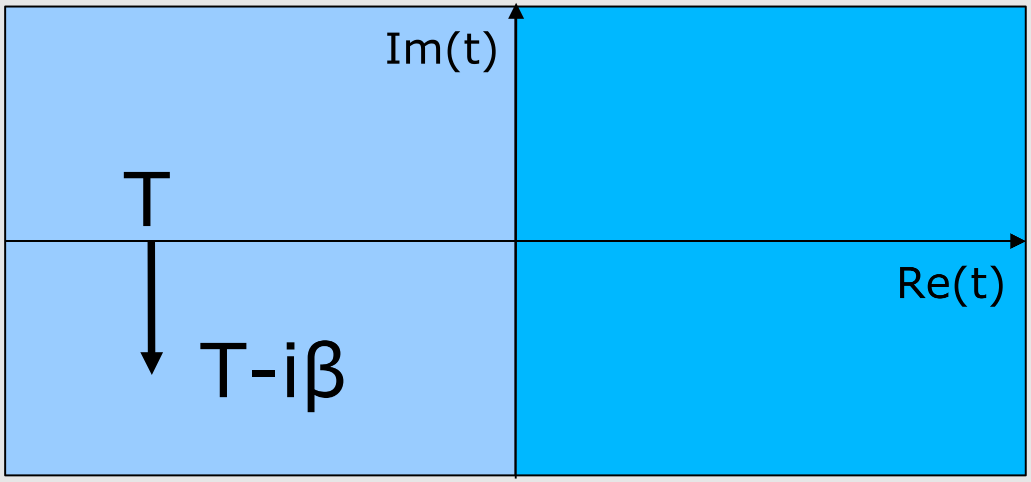
Closed time path formalism

$$H(t) = H_i, \Re(t) \leq 0$$

Equilibrium state, β

$$H(t) = H(t), \Re(t) > 0$$

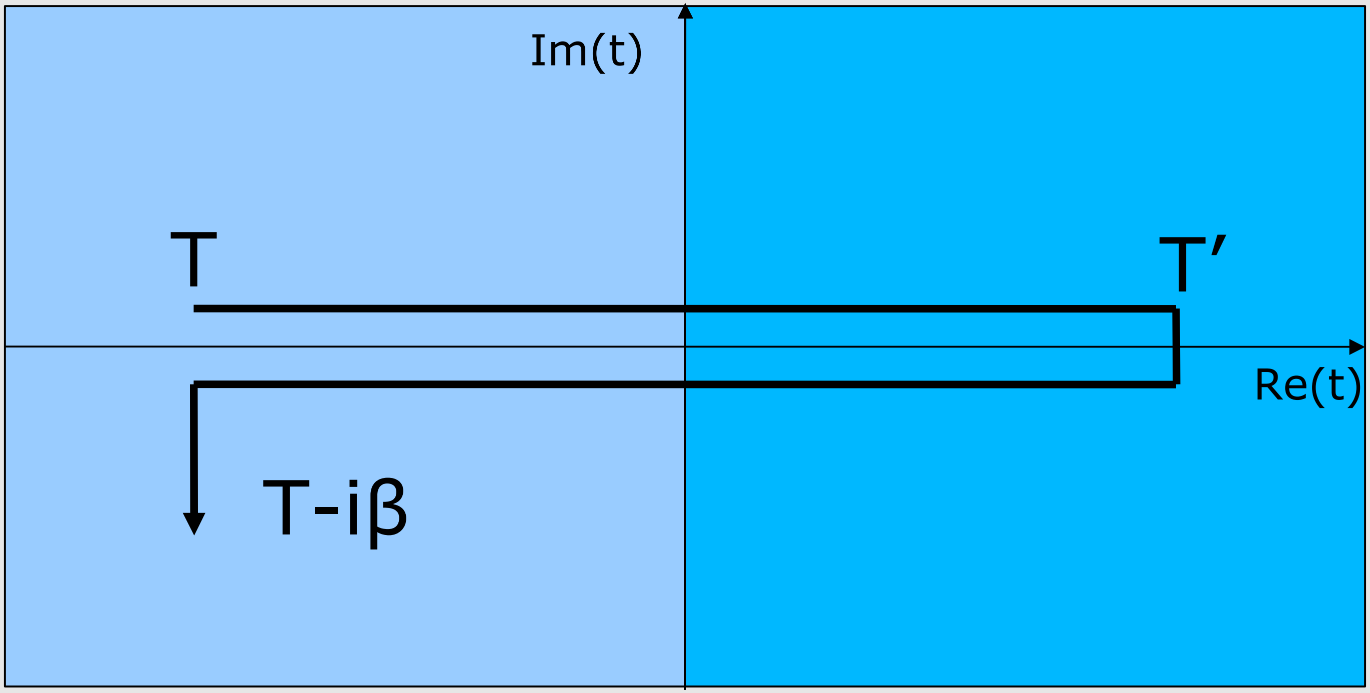
$$\rho(0) = \frac{e^{-\beta H_i}}{Tr e^{-\beta H_i}} = \frac{U(T - i\beta, T)}{Tr U(T - i\beta, T)}$$



Closed time path formalism

$$\langle A \rangle (t) = Tr\rho(t)A$$

$$= \frac{TrU(T-i\beta,T)U(T,T')U(T',t)AU(t,T)}{TrU(T-i\beta,T)U(T,T')U(T',T)}$$



Closed time path formalism

Define generating functional as:

$$Z\left[J_c\right]=TrU_{J_c}\left(T-i\beta,T\right)U_{J_c}\left(T,T'\right)U_{J_c}\left(T',T\right)$$

Or equivalently, setting T, T′ to infinity:

$$Z\left[J_c\right]=\int D\varphi e^{i\int_C dt\int d^3x\left(L+J_c\varphi\right)}$$

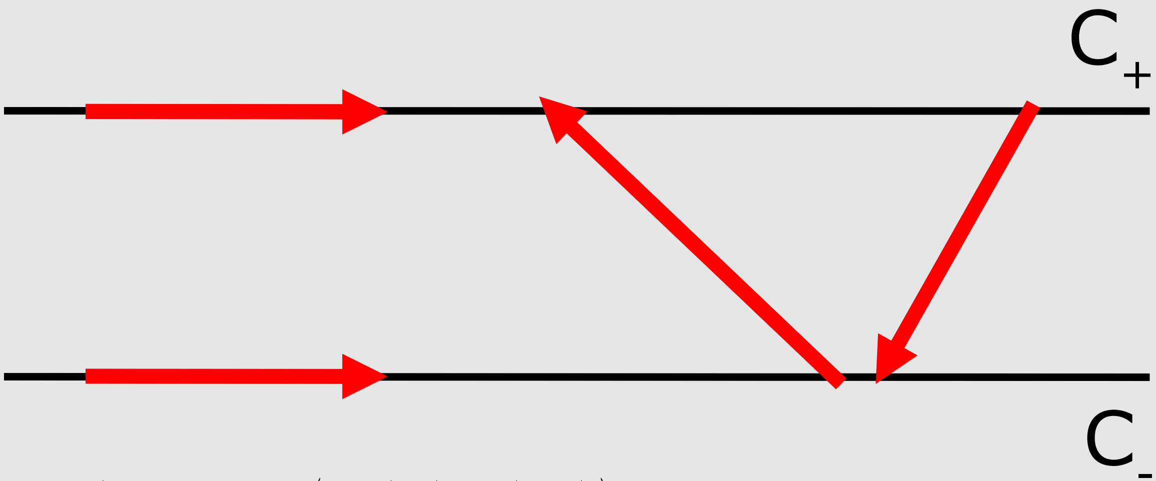
T_c ordered Green's functions

$$G_c\left(t-t'\right)$$

via functional derivative

Green's functions

Four propagator structures are possible:



$$iG_{++}\left(t-t'\right) = \left\langle T\left(\varphi\left(t\right)\varphi\left(t'\right)\right)\right\rangle$$

$$iG_{+-}\left(t-t'\right) = \left\langle \varphi\left(t'\right)\varphi\left(t\right)\right\rangle$$

$$iG_{-+}\left(t-t'\right) = \left\langle \varphi\left(t\right)\varphi\left(t'\right)\right\rangle$$

$$iG_{--}\left(t-t'\right) = \left\langle T_{inv}\left(\varphi\left(t\right)\varphi\left(t'\right)\right)\right\rangle$$

anti time ordered!

Green's functions

Double fields and sources:

$$\varphi \rightarrow \left(\varphi_+ ,\varphi_-\right)$$

$$J \rightarrow \left(J_+ ,J_-\right)$$

The action becomes:

$$S = \int d^4 x \left[L\left(\varphi_+ ,J_+\right) - L\left(\varphi_- ,J_-\right)\right]$$

Calculate G in the “standard” fashion:

$$iG_{ab}\left(x,y\right) = \left(-i\right)^2 \frac{1}{Z} \frac{\delta^2 Z\left[J\right]}{\delta J_a\left(x\right)\delta J_b\left(y\right)} \Bigg|_{J=0}$$

Green's functions

Physical Green’s functions in momentum space:

$$G_R(x,x') = \theta(t-t')\langle[\varphi(x),\varphi(x')]\rangle = \frac{1}{p^2-m^2+i\varepsilon p^0}$$

$$G_A(x,x') = \theta(t'-t)\langle[\varphi(x),\varphi(x')]\rangle = \frac{1}{p^2-m^2-i\varepsilon p^0}$$

$$G_C(x,x') = \langle[\varphi(x),\varphi(x')]_+\rangle = -2\mathrm{i}\pi\left(1+2\mathrm{n}_B(\omega_p)\right)\delta\left(p^2-m^2\right)$$

G_R and G_A are T-independent at lowest order!

Green's functions

And in Nonequilibrium?

Simply replace

$$n_B \rightarrow f_B$$

in the bare propagators

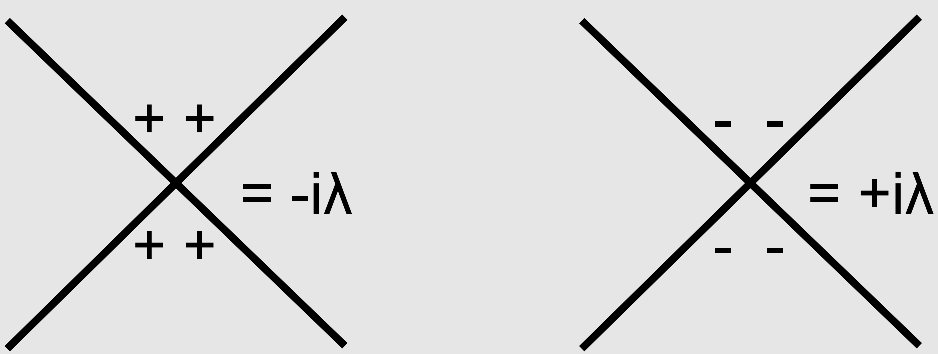
Propagators (one loop)

Consider φ^4 theory:

$$L(\varphi) = \frac{1}{2}\partial_\mu \varphi \,\partial^\mu \varphi - \frac{m^2}{2}\varphi^2 - \frac{\lambda}{4!}\varphi^4$$

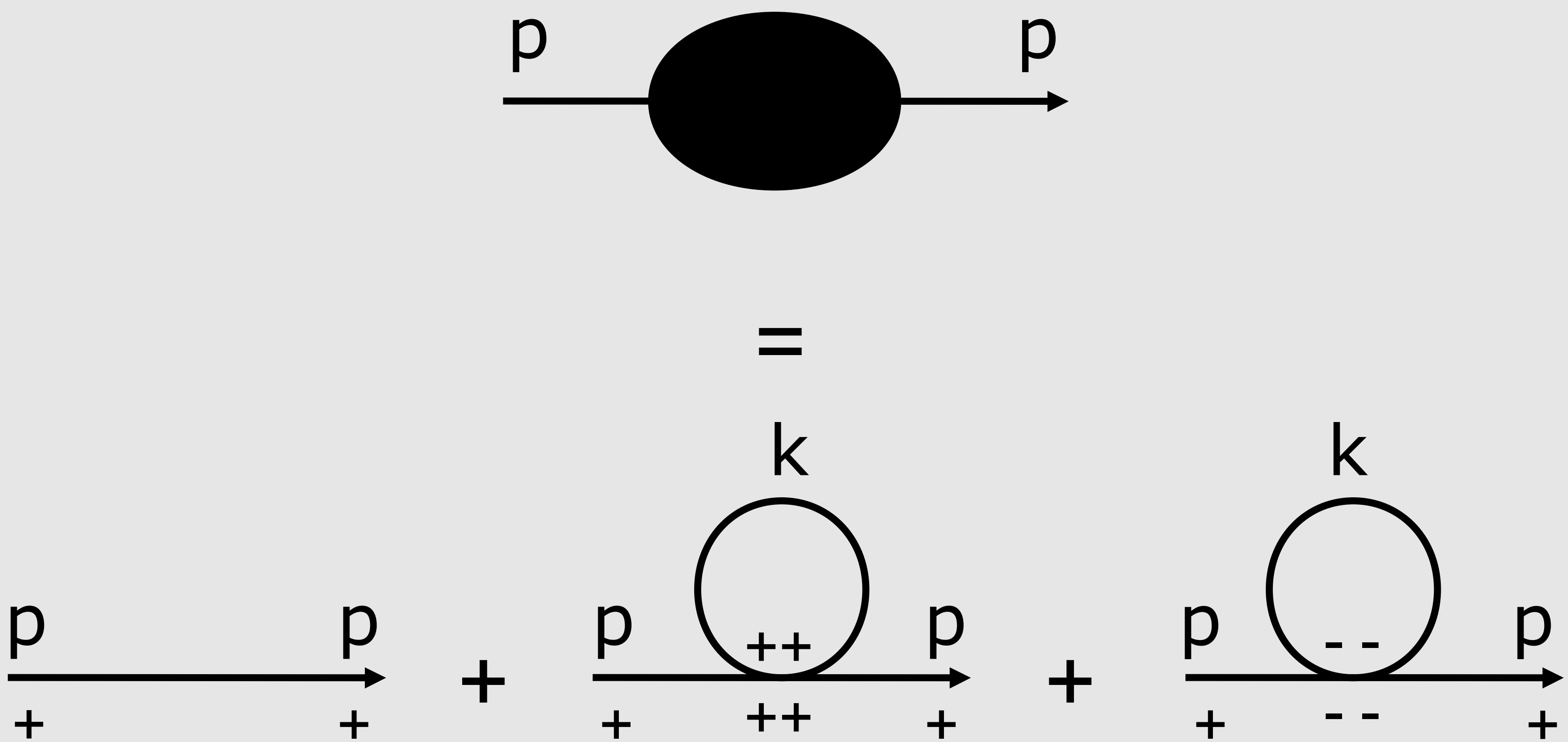
$$S[\varphi_+, \varphi_-] = \int d^4x \left[L(\varphi_+) - L(\varphi_-) \right]$$

Two interaction vertices:



Propagators (one loop)

Calculate one loop correction:



Applications

- Domain growth
- Dissipation and fluctuation

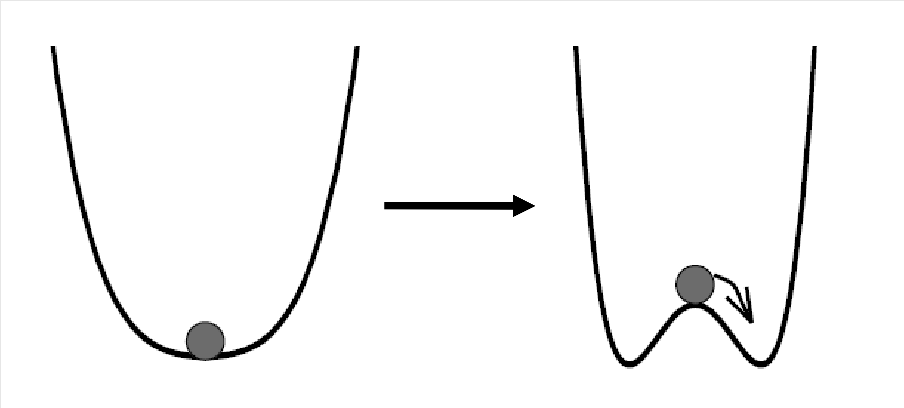
Domain growth

Consider nonequilibrium σ-model

$$L=\frac{1}{2}\partial_\mu\varphi_a\partial^\mu\varphi_a-\frac{1}{2}m_i^2\varphi_a\varphi_a+\frac{1}{2}\delta m^2(t)\varphi_a\varphi_a-\frac{\lambda}{4}(\varphi_a\varphi_a)^2$$

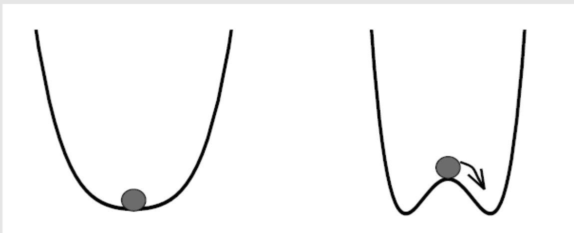
With time dependent mass squared

$$\delta m^2(t)=\theta(t)\left(m_i^2+m_f^2\right)$$



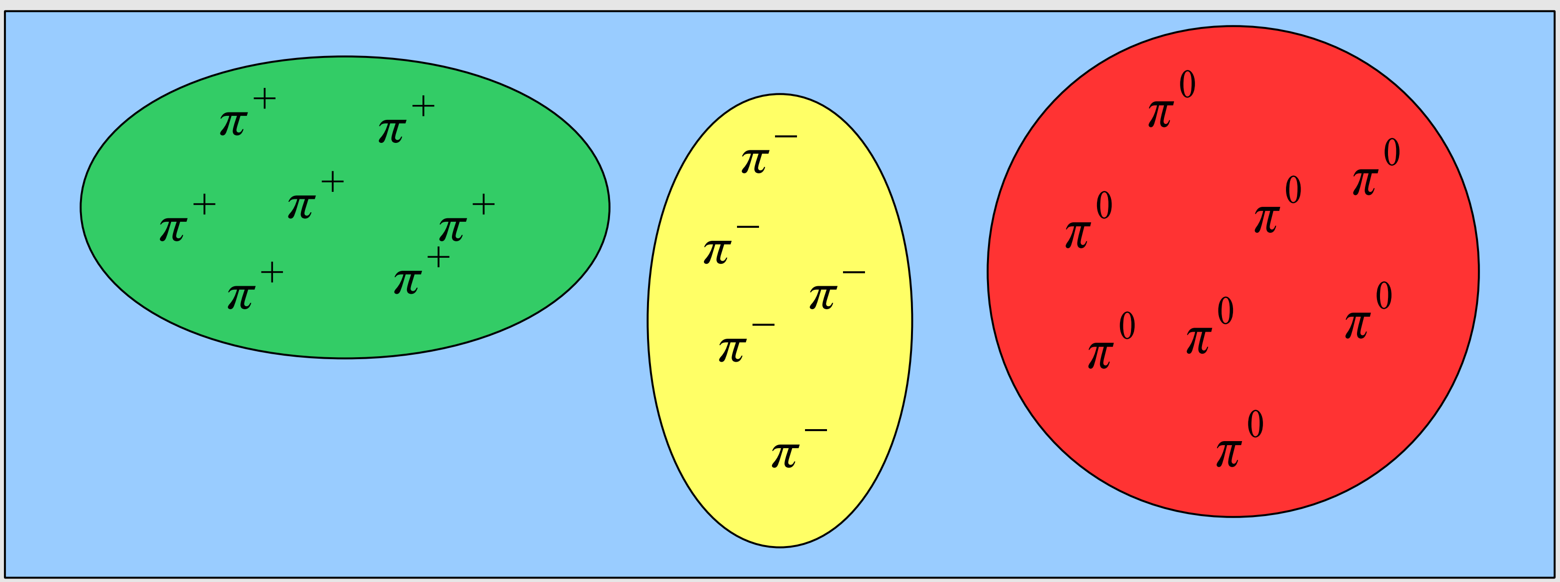
quench at t=0

Domain growth



quench at t=0

Instability creates long-range correlation,
Disoriented Chiral Condensate



Domain growth

Aim: calculate time evolution of σ-/π-correlated regions:

$$\langle \sigma(t,\vec{x})\sigma(t,0)\rangle$$

For NJL model, 1/N-expansion:

$$G_C(t,|\vec{x}|)=C(T)\left(\frac{|\vec{x}|}{t}\right)^{\alpha} e^{Mt} e^{\frac{-|\vec{x}|^2}{L^2(t)}}$$

With typical domain size

$$L(t)=\sqrt{\frac{b}{2\mathrm{M}}t}$$

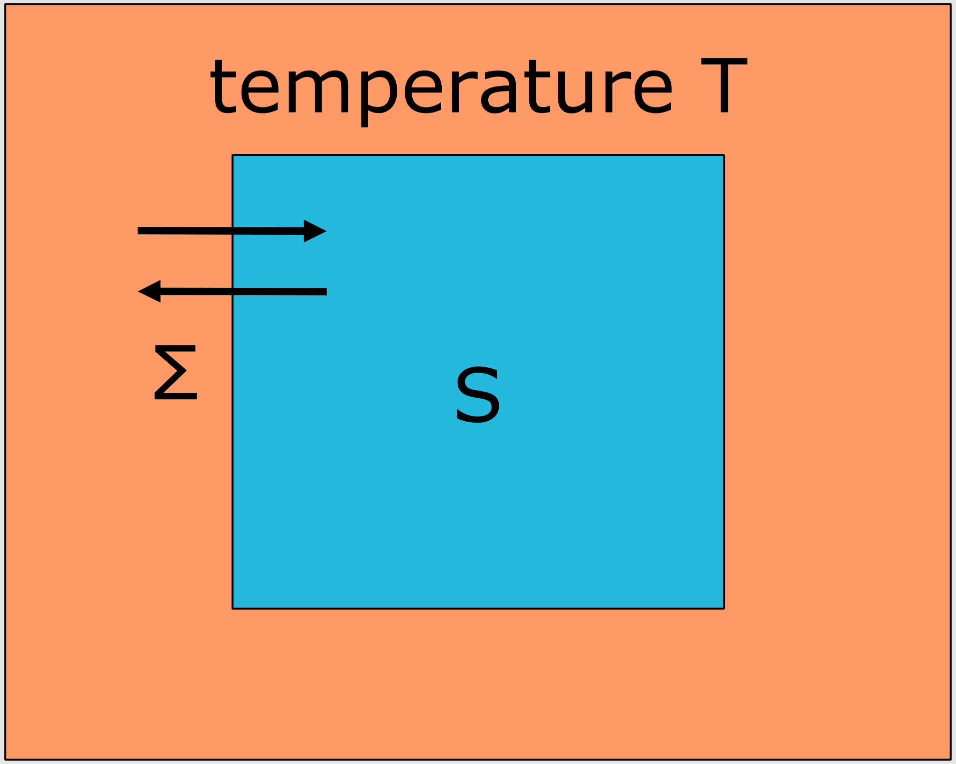
Domain growth

$$L(t) = \sqrt{\frac{b}{2\text{M}}}\, t$$

- Domain size grows linearly with time
- Plasma produces large number of coherent pions
- Indefinite growth due to lack of collisions

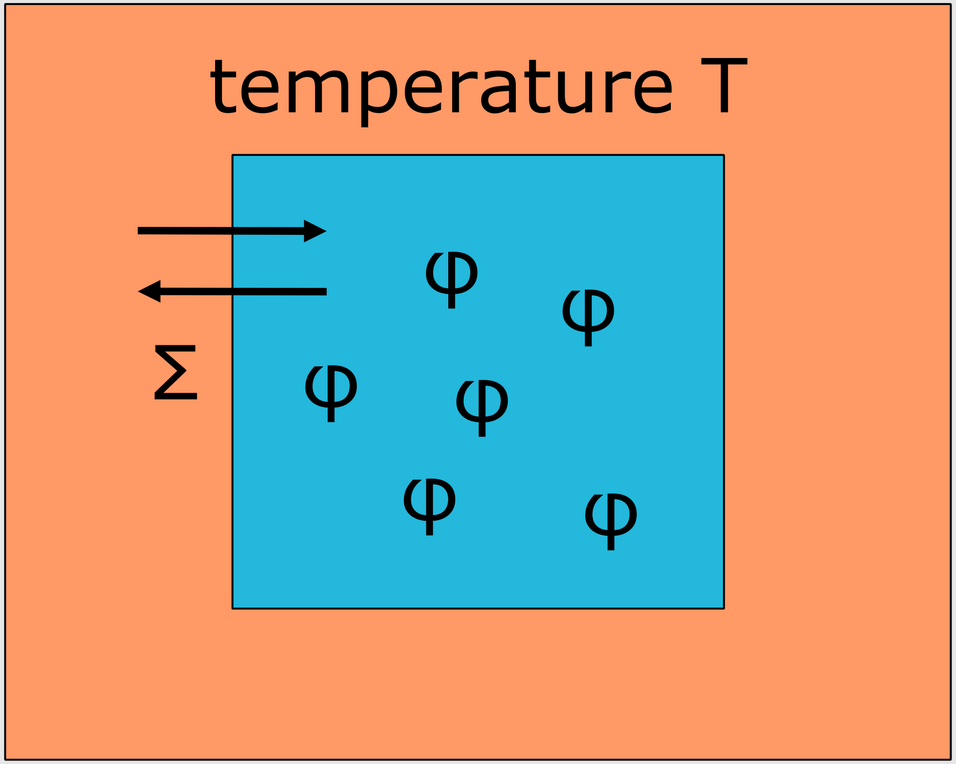
Fluctuation and dissipation

Quantum mechanical system S
interacting with heat bath at temperature T



Fluctuation and dissipation

$$S = \frac{1}{2}\left[\phi_c\left(-\partial_\mu \partial^\mu - m^2\right)\phi_c - \phi_c \Sigma^c \phi_c\right]$$



Fluctuation and dissipation

Equations of motion for the GFs yield

$$\left(-\partial_\mu \partial^\mu - m^2\right) G_< - \Sigma_R G_< - \Sigma_< G_A = 0$$

Kadanoff-Baym equation

Determines complete and causal non-equilibrium evolution for the two-point functions

Fluctuation and dissipation

Linear combination of $\Sigma_<$ and $\Sigma_>$ gives

$s\left(x_{1,}x_2\right)$	mass shift
$a\left(x_{1,}x_2\right)$	dissipation term
$I\left(x_{1,}x_2\right)$	fluctuation term

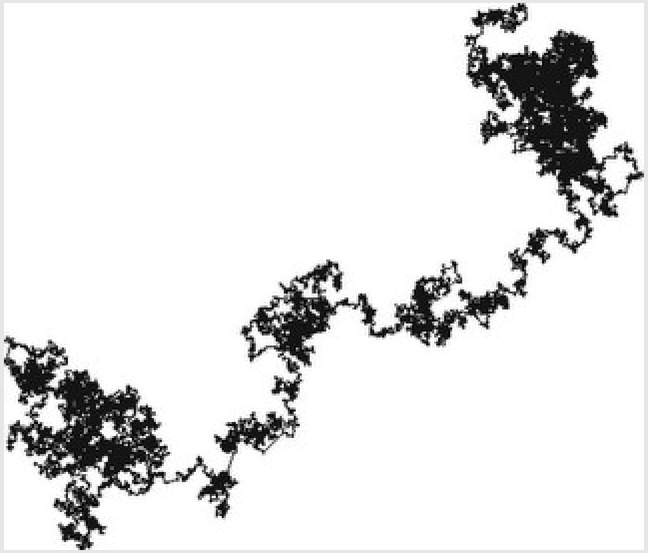
Fluctuation and dissipation

Fourier transformed stochastic EoM for φ_ξ

$$\ddot{\phi}_\xi(k,t)+(k^2+m^2+s)\phi_\xi(k,t)+2\int_{-\infty}^{t}dt'\,\Gamma(t-t')\phi_\xi(t')=\xi(k,t)$$

Cf. Brownian motion

$$M\ddot{x}+M\omega^2x+2\int dt'\,\Gamma(t-t')\dot{x}(t')=\xi(t)$$

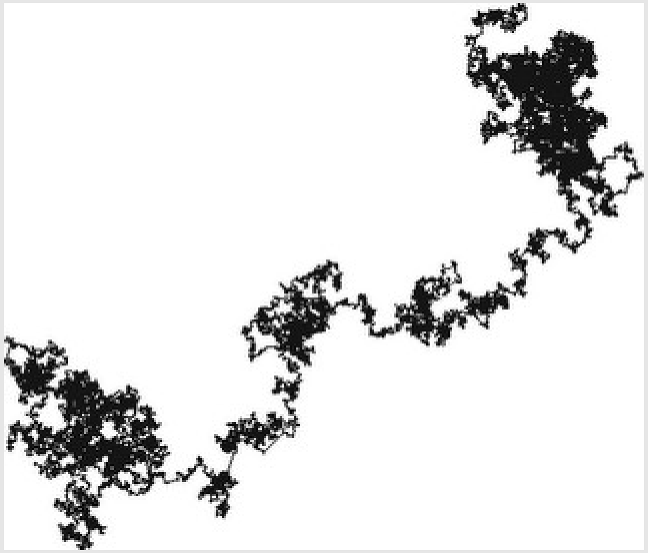


Fluctuation and dissipation

Generalized (microscopic)
Fluctuation-Dissipation-Theorem (high T limit)

$$I(k) = \frac{T}{k_0} 2\mathrm{i}a(k)$$

- Noise term „heats" system
- Dissipative term counteracts
- Modes thermally populated
- System will thermalize at T



References

V. Koch: „*Aspects of chiral symmetry*" (2008)

A. Das: „*Finite temperature field theory*", World Scientific (1997)

E. A. Calzetta, B. Hu: „*Nonequilibrium quantum field theory*", Cambridge Monographs on mathematical physics (2008)

C. Greiner, S. Leupold: „*Stochastic interpretation of Kadanoff-Baym Equations and their relation to Langevin processes*" (1998)

Thank you